Hf 1, Definíció alapján $f'(a) = ?$

a) $f(x) = \frac{1}{x^2}$ $(x<0)$ $a=-1$,

$$f'(-1) = \lim_{h\to 0} \frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0} \frac{\frac{1}{(-1+h)^2} - 1}{h} =$$

$$= \lim_{h\to 0} \frac{1-(h-1)^2}{h(h-1)^2} = \lim_{h\to 0} \frac{1-(h^2-2h+1)}{h(h-1)^2} = \lim_{h\to 0} \frac{2h-h^2}{h(h-1)^2} =$$

$$= \lim_{h\to 0} \frac{2-h}{(h-1)^2} = \underline{\underline{2}}$$

b) $f(x) = \begin{cases} x^3+x, & \text{ha } x\le 0, \\ e^x-1, & \text{ha } x>0, \end{cases}$ $a=0$,

$$f'_-(0) = \lim_{h\to 0-0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0-0} \frac{(0+h)^3+(0+h)-0}{h} =$$

$$= \lim_{h\to 0-0} \frac{h^3+h}{h} = \lim_{h\to 0-0} (h^2+1) = 1$$

$$f'_+(0) = \lim_{h\to 0+0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0+0} \frac{(e^{0+h}-1)-0}{h} =$$

$$= \lim_{h\to 0+0} \frac{e^h-1}{h} = 1 \quad \text{(nevezetes határérték)}$$

$$f'_-(0) = f'_+(0) \Rightarrow \exists\, f'(0) = \underline{\underline{1}}.$$

2, Deriválni!

a) $f(x) = x^2 e^{\cos x}$ $(x\in\mathbb{R})$

$$f'(x) = \underbrace{(x^2)'}_{2x} e^{\cos x} + x^2 \underbrace{(e^{\cos x})'}_{e^{\cos x}\underbrace{(\cos x)'}_{-\sin x}} = 2x e^{\cos x} - x^2 e^{\cos x} \cdot \sin x$$

b) $f(x) = \log_2\left(\frac{x+2}{x-1}\right)$ $(x>1)$

$$f'(x) = \frac{1}{\frac{x+2}{x-1}\cdot \ln 2} \cdot \left(\frac{x+2}{x-1}\right)' = \frac{\cancel{x-1}}{x+2} \cdot \frac{(x+2)'(x-1)-(x+2)(x-1)'}{(x-1)^{\cancel{2}} \ln 2} =$$

$$= \frac{1\cdot(x-1)-(x+2)\cdot 1}{(x+2)(x-1)\ln 2} = \frac{-3}{(x+2)(x-1)\ln 2}$$

c) $f(x) = \sin\sqrt{2^x + x^2} \quad (x \in \mathbb{R})$

$$f'(x) = \cos\sqrt{2^x+x^2} \cdot \underbrace{\left(\sqrt{2^x+x^2}\right)'}_{\frac{1}{2\sqrt{2^x+x^2}} \cdot \underbrace{(2^x+x^2)'}_{2^x \ln 2 + 2x}} =$$

$$= \cos\sqrt{2^x+x^2} \cdot \frac{2^x \ln 2 + 2x}{2\sqrt{2^x+x^2}}$$

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3 a) $f(x) = x^x \quad (x>0)$

$$f(x) = \left(e^{\ln x}\right)^x = e^{x \ln x}$$

$$f'(x) = e^{x\ln x} \cdot \underbrace{(x \ln x)'}_{1 \cdot \ln x + x \cdot \frac{1}{x}} = x^x(\ln x + 1)$$

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3 b) $f(x) = (x^3+x)^{\ln x} \quad (x>1)$

$$f(x) = \left(e^{\ln(x^3+x)}\right)^{\ln x} = e^{\ln x \cdot \ln(x^3+x)}$$

$$f'(x) = e^{\ln x \cdot \ln(x^3+x)} \cdot \underbrace{\left(\ln x \cdot \ln(x^3+x)\right)'}_{\frac{1}{x}\ln(x^3+x) + \ln x \cdot \frac{1}{x^3+x} \cdot (3x^2+1)}$$

$$= (x^3+x)^{\ln x}\left[\frac{\ln(x^3+x)}{x} + \frac{\ln x\,(3x^2+1)}{x^3+x}\right]$$

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2 d) $f(x) = \dfrac{\cos(\ln 2x)}{x^2 \ln x} \quad (x>0)$

$$f'(x) = \frac{[\cos(\ln 2x)]' x^2 \ln x - \cos(\ln 2x)\,[x^2 \ln x]'}{(x^2 \ln x)^2}$$

$$= \frac{-\sin(\ln 2x)\frac{1}{x} \cdot x^2 \ln x - \cos(\ln 2x) \cdot (2x\ln x + x)}{(x^2 \ln x)^2}$$

hiszen

$$[\cos(\ln 2x)]' = -\sin(\ln 2x) \cdot \underbrace{(\ln 2x)'}_{\frac{1}{2x} \cdot 2} = -\sin(\ln(2x)) \cdot \frac{1}{x}$$

$$[x^2 \cdot \ln x]' = 2x \cdot \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x$$