HF

1a) $\displaystyle\int \frac{(x+1)^2}{\sqrt{x}}\,dx = \int \frac{x^2+2x+1}{x^{1/2}}\,dx = \int x^{3/2} + 2x^{1/2} + x^{-1/2}\,dx =$

(x>0)

$\displaystyle = \frac{x^{5/2}}{5/2} + 2\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{2}{5}\sqrt{x^5} + \frac{4}{3}\sqrt{x^3} + 2\sqrt{x} + C$

1b) $\displaystyle\int \sqrt{1-\cos 2x}\,dx = \int \sqrt{\cos^2 x + \sin^2 x - (\cos^2 x - \sin^2 x)}\,dx =$

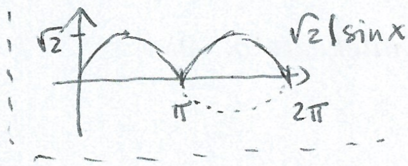
$(0<x<2\pi)$

$\displaystyle = \int \sqrt{2\sin^2 x}\,dx = \sqrt{2}\int |\sin x|\,dx = \begin{cases} -\sqrt{2}\cdot\cos x + c_1 & (0<x\le\pi) \\ \sqrt{2}\cos x + c_2 & (\pi<x<2\pi) \end{cases} = (*)$

$c_1, c_2 = ?$ $\rightarrow$ a primitív függvények folytonosak a π pontban.

$\cos\pi = -1 \Rightarrow \sqrt{2} + c_1 = -\sqrt{2} + c_2 \Rightarrow c_1 = -2\sqrt{2} + c_2$ $\quad (c := c_2)$

$\displaystyle (*) = \begin{cases} -\sqrt{2}\cos x - 2\sqrt{2} + c & (0<x\le\pi) \\ \sqrt{2}\cos x + c & (\pi<x<2\pi) \end{cases}$



1c) $\displaystyle\int \frac{1}{1+e^{-x}}\,dx = \int \frac{e^x}{e^x+1}\,dx = \left(\frac{f'}{f}\text{ típus}\right) = \ln(e^x+1) + C$

$(x\in\mathbb{R})$

1d) $\displaystyle\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx = \left(\frac{f'}{f}\text{ típus}\right) = \frac{1}{2}\ln(x^2+4) + C$

$(x\in\mathbb{R})$

1e) $\displaystyle\int \frac{x}{\sqrt[3]{x^2+4}}\,dx = \frac{1}{2}\int 2x(x^2+4)^{-1/3}\,dx = (f^\alpha\cdot f'\text{ típus}) =$

$(x\in\mathbb{R})$

$\displaystyle = \frac{1}{2}\frac{(x^2+4)^{2/3}}{2/3} + C = \frac{3}{4}\sqrt[3]{(x^2+4)^2} + C$

1h) $\displaystyle\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int \frac{2x}{1+(x^2)^2}\,dx = \left(\frac{1}{1+f^2}\cdot f'\text{ típus}\right) =$

$(x\in\mathbb{R})$

$\displaystyle = \frac{1}{2}\cdot \operatorname{arctg} x^2 + C$

1f) $\displaystyle\int x^2\sqrt[3]{6x^3+4}\,dx = \frac{1}{18}\int 18x^2(6x^3+4)^{\frac{1}{3}}\,dx = (f^\alpha\cdot f'\text{ típus}) =$

$(x\in\mathbb{R})$

$\displaystyle = \frac{1}{18}\frac{(6x^3+4)^{\frac{4}{3}}}{4/3} + C = \frac{3}{72}\sqrt[3]{(6x^3+4)^4} + C$

1 g) $\int \frac{5x+3}{2x-3}\,dx = \int \frac{5}{2} + \frac{21}{2}\cdot\frac{1}{2x-3}\,dx =$
$(x>3/2)$

$5x+3 : 2x-3 = 5/2$
$-\;(5x-\frac{15}{2})$
$\frac{21}{2}$

$= \frac{5}{2}x + \frac{21}{2}\,\frac{\ln(2x-3)}{2} + C$

$= \frac{5}{2}x + \frac{21}{4}\ln(2x-3) + C$

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1 i) $\int x\ln^2 x\,dx = \int \left(\frac{x^2}{2}\right)'\ln^2 x\,dx = \frac{x^2}{2}\ln^2 x - \int \frac{x^2}{2}\cdot 2\ln x\cdot\frac{1}{x}\,dx =$
$(x>0)$

$= \frac{x^2}{2}\ln^2 x - \int x\ln x\,dx + C \;= (*)$

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$\int x\ln x\,dx = \int\left(\frac{x^2}{2}\right)'\ln x\,dx = \frac{x^2}{2}\ln x - \int\frac{x^2}{2}\cdot\frac{1}{x}\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx =$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

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$(*) = \frac{x^2}{2}\ln^2 x - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C$

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1 j) $I = \int e^x\sin(3x+1)\,dx = \int (e^x)'\sin(3x+1)\,dx =$
$(x\in\mathbb{R})$

$= e^x\sin(3x+1) - \int e^x\cdot\cos(3x+1)\cdot 3\,dx =$

$= e^x\sin(3x+1) - 3\int e^x\cos(3x+1)\,dx = (*)$

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$\int e^x\cos(3x+1)\,dx = \int (e^x)'\cos(3x+1)\,dx =$

$= e^x\cos(3x+1) - \int e^x(-\sin(3x+1)\cdot 3)\,dx =$

$= e^x\cos(3x+1) + 3\int e^x\sin(3x+1)\,dx = e^x\cos(3x+1) + 3I$

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$(*) = e^x\sin(3x+1) - 3\left[e^x\cos(3x+1) + 3I\right] =$

$= e^x\sin(3x+1) - 3e^x\cos(3x+1) - 9I$

$\Rightarrow\; 10I = e^x\cdot\sin(3x+1) - 3e^x\cos(3x+1) + C$

$\int e^x\sin(3x+1)\,dx = \frac{e^x}{10}\left(\sin(3x+1) - 3\cos(3x+1)\right) + C$