Hf 1, L'Hospital szabály:

a) $$\lim_{x\to\frac{\pi}{2}-0} \frac{\ln\left(\frac{\pi}{2}-x\right)}{\operatorname{tg} x} = \left(\frac{-\infty}{+\infty}\right) \overset{L'H}{=} \lim_{x\to\frac{\pi}{2}-0} \frac{\frac{1}{\frac{\pi}{2}-x}(-1)}{\frac{1}{\cos^2 x}} =$$

$$= \lim_{x\to\frac{\pi}{2}-0} \frac{\cos^2 x}{x-\frac{\pi}{2}} = \left(\frac{0}{0}\right) \overset{L'H}{=} \lim_{x\to\frac{\pi}{2}-0} \frac{2\cos x(-\sin x)}{1} = \frac{2\cdot 0\cdot(-1)}{1} = \underline{\underline{0}}$$

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b) $$\lim_{x\to 0+0} \frac{1-\sqrt{\cos x}}{1-\cos\sqrt{x}} = \left(\frac{0}{0}\right) \overset{L'H}{=} \lim_{x\to 0+0} \frac{-\frac{1}{2\sqrt{\cos x}}\cdot(-\sin x)}{\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}}} =$$

$$= \lim_{x\to 0+0} \frac{1}{\sqrt{\cos x}} \cdot \lim_{x\to 0+0} \frac{x\cdot\frac{\sin x}{x}}{\frac{\sin\sqrt{x}}{\sqrt{x}}} = 1\cdot\frac{0\cdot 1}{1} = \underline{\underline{0}}$$

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c) $$\lim_{x\to+\infty} x\cdot\left(\operatorname{arctg} x-\frac{\pi}{2}\right) = \lim_{x\to+\infty} \frac{\operatorname{arctg} x-\frac{\pi}{2}}{\frac{1}{x}} = \left(\frac{0}{0}\right) \overset{L'H}{=}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x\to+\infty} -\frac{x^2}{1+x^2} = -\lim_{x\to+\infty} \frac{1}{\frac{1}{x^2}+1} = -\frac{1}{0+1} = \underline{\underline{-1}}$$

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d) $$\lim_{x\to 0} (\operatorname{ch} x)^{1/\operatorname{sh} x} = \left(1^{\pm\infty}\right)$$

$$(\operatorname{ch} x)^{1/\operatorname{sh} x} = \left(e^{\ln\operatorname{ch} x}\right)^{1/\operatorname{sh} x} = e^{\frac{\ln\operatorname{ch} x}{\operatorname{sh} x}}$$

$$\lim_{x\to 0} \frac{\ln\operatorname{ch} x}{\operatorname{sh} x} = \left(\frac{0}{0}\right) \overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{\operatorname{ch} x}\cdot\operatorname{sh} x}{\operatorname{ch} x} = \lim_{x\to 0} \frac{\operatorname{sh} x}{\operatorname{ch}^2 x} = \frac{0}{1^2} = 0$$

Mivel exp folytonos a 0 pontban, így

$$\lim_{x\to 0} (\operatorname{ch} x)^{1/\operatorname{sh} x} = e^{\lim\limits_{x\to 0}\frac{\ln\operatorname{ch} x}{\operatorname{sh} x}} = e^0 = \underline{\underline{1}}$$

2. Konvexitás

a) $f(x) = e^{2x} - (4x+1) \quad (x \in \mathbb{R})$

$f'(x) = 2 \cdot e^{2x} - 4 \quad , \quad f''(x) = 4e^{2x} > 0 \implies f$ konvex $\mathbb{R}$-en.

b) $f(x) = \dfrac{4x}{x^2-1} \quad (x \neq \pm 1)$

$$f'(x) = \frac{4(x^2-1) - 4x \cdot 2x}{(x^2-1)^2} = -\frac{4x^2+4}{(x^2-1)^2}$$

$$f''(x) = -\frac{8x(x^2-1)^{\cancel{2}} - (4x^2+4) \cdot 2\cancel{(x^2-1)} \cdot 2x}{(x^2-1)^{\cancel{4}\,3}} =$$

$$= -\frac{8x^3 - 8x - 16x^3 - 16x}{(x^2-1)^3} = \frac{8x^3 + 24x}{(x^2-1)^3} = \frac{8x(x^2+3)}{(x^2-1)^3}$$

$f''(x) = 0 \iff x = 0$

	$x<-1$	$-1<x<0$	0	$0<x<1$	$x>1$
f''	$-$	$+$	0	$-$	$+$
f	$\cap$	$\cup$	infl.	$\cap$	$\cup$

3. Asszimptota:

$$f(x) = \frac{x^2+4}{x} \quad (x \neq 0)$$

$$\lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{x^2+4}{x^2} = \lim_{x \to \pm\infty} \left(1 + \frac{4}{x^2}\right) = 1 = A$$

$$\lim_{x \to \pm\infty} (f(x) - Ax) = \lim_{x \to \pm\infty} \left(\frac{x^2+4}{x} - x\right) = \lim_{x \to \pm\infty} \left(\frac{x^2+4-x^2}{x}\right) =$$

$$= \lim_{x \to \pm\infty} \frac{4}{x} = 0 = B.$$

Van asszimptotája $(+\infty)$-ben $\to y = x$

és $(-\infty)$-ben $\to y = x$.